A typical 16th-century accounting book from Italy held together by a tacket hand-sewn binding. Sources like this from the special collections of Tel Aviv University's Central library and numerous other research libraries around the world convey important clues about the social, cultural, cognitive and intellectual lives of early-modern merchants

By Simon Lewsen
Photographs by Boaz Perlstein

THINK AGAIN

An examination of the evolution of mathematical practices reveals that accepted accounts of the great cognitive shift might not add up

There's a familiar story about the history of modernity and mathematics — a story that Ray Schrire, a historian and Azrieli Early Career Faculty Fellow at Tel Aviv University, believes is mostly wrong. It goes like this: in premodern Europe, people practised premodern math. Merchants and shopkeepers managed their books with Roman numerals, but these weren't numbers, at least as we understand them. They were more like codes or coordinates — a set of instructions that would tell you how to move beads around on an abacus or slide a token around a table.

For the average bookkeeper, mathematics was less a system of abstract thought than a set of practices. If you kept diligent records and used your abacus well, you could perform basic calculations and get the results you were seeking. But people had limited insights into what they were doing: *performing* calculations is different from *understanding* them. And medieval practitioners couldn't get their heads around exponents, infinite series or negative numbers. Tellingly, there is no Roman numeral for zero. Why would there be? You can't move zero beads from one abacus rung to another.

In the early-modern era (the period beginning in the late 15th century and ending with the dawn of the 18th century), everything changed, or so the story goes. Europeans abandoned Roman numerals for the Hindu-Arabic notational system we still use today, and they adopted written arithmetic, a practice imported from the Middle East. Some of the smartest thinkers in the West — Isaac Newton, Blaise Pascal, Gottfried Leibniz — began pushing mathematics into abstract territory, inventing entire disciplines that hadn't existed before. And thanks to the advent of global capitalism, members of the European professional classes suddenly found

themselves calculating dividends, compound interest rates and insurance premiums. These new ideas quickly proliferated, making ordinary working people more sophisticated, more capable of abstract thought — in a word, more modern — than their medieval forebears had been. A commercial and industrial revolution gave rise to a cognitive one.

Or did it? Schrire isn't convinced. He has studied the documents left behind by early-modern professionals — poring over account books and bills of sale from merchants, shopkeepers, land surveyors and notaries — and been surprised by what he found. Or rather, what he *didn't* find.

If the advent of capitalism and industrial modernity really led to the advent of modern mathematics, not just among Europe's brightest minds but among its rank-and-file practitioners too, you'd expect written proof. You'd expect, for instance, to see evidence that everyday professionals were practising double-entry bookkeeping, a modern system of accounting where transactions are entered twice, once as a debit and once as a credit, to safeguard against arithmetic error. You'd expect to see records kept in Hindu-Arabic numerals rather than their Roman predecessors. And you'd expect to find seemingly endless piles of scrap paper on which bookkeepers, having abandoned the abacus, had written out their calculations in longhand.

Schrire is not yet sure what to make of his finding. But it may have profound implications for how we understand European history — and even how we understand ourselves in the present day.

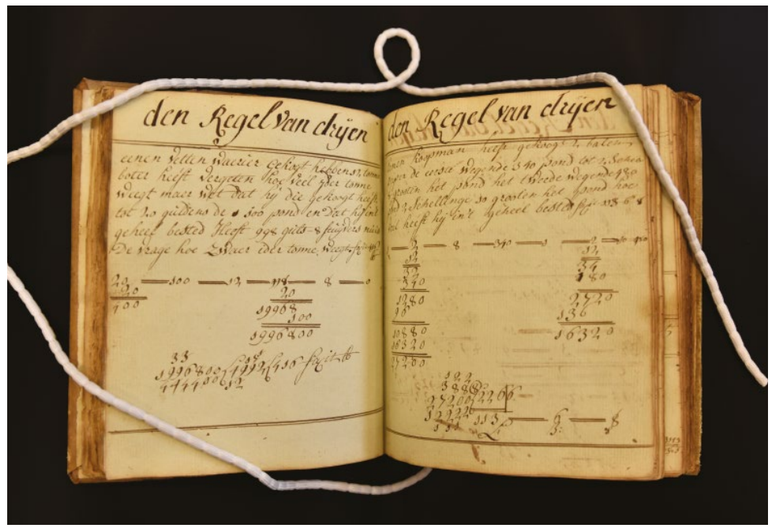
But Schrire hasn't seen any of this, at least not in sufficient quantities. "When we think about capitalism, we think about people doing stuff with numbers," he says. "A precondition for capitalism is that individuals are rational and can calculate the best ways of increasing their utility. But my research suggests that people simply weren't doing the things that historians say they should have done." He's not yet sure what to make of this finding. But it may have profound implications for how we understand European history — and even how we understand ourselves in the present day.

Schrire discovered his scholarly interests mostly by accident. As a master's student at the Hebrew University of Jerusalem, he attended a talk by Ayelet Even-Ezra, a professor in the history department who had reviewed late-medieval manuscripts and noticed the proliferation of horizontal tree diagrams: flow charts that map out a set of ideas via a series of forking paths.

Even-Ezra believed that these drawings, which hadn't appeared before the 12th century, could give us a glimpse into the late-medieval mind at work. Novel visualization techniques had surely given rise to novel modes of thought. Thanks to the advent of tree diagrams, she argued, ancient scholars had practised new forms of counter-factual reasoning, which they applied to theology, law and natural sciences. Schrire was fascinated by Even-Ezra's talk. She was doing the same basic task that all historians did — looking closely at archival documents — but she was asking questions that belonged more to cognitive science than to history: how do people think? How do their modes of cognition determine their understandings of the world? "I went to her office," Schrire recalls, "and basically said, 'I want to do whatever you're doing.'"

He decided to devote his graduate research to book history and human thought, although he didn't have much of a game plan. During the last year of his MA and the first two years of his PhD, he split his time between Israel and California's Bay Area, with his partner (now wife) Ella Elbaz, then a PhD student in the Department of Comparative Literature at Stanford University. In the United States, he frequented the rare books collections at the University of California, Berkeley. "I had no idea what I was doing," he says. "I would page up old books at random, hoping for insights into cognition."

Tel Aviv University historian and Azrieli Early Career Faculty Fellow Ray Schrire carefully scrutinizes an early-modern accounting book for traces of arithmetical practice exposed through the types of numerals used and kinds of calculation errors made.



A 17th-century student's notebook from the Netherlands (above right) attests to the types of training received in early-modern business schools, as well as to the performative aspect of bookkeeping. A reckoning token minted in 16th-century Nuremberg (above left) that was used for performing actual calculations across Europe. The image on the token shows an early-modern merchant engrossed in such arithmetical activity. These and other sources (top) suggest to Schrire "that people simply weren't doing the things that historians of mathematics say they should have done."

Soon, he found an intriguing specimen: a classic Latin grammar textbook that had been used in Britain from the Middle Ages to the early 19th century. The owner of this particular copy, a 17th-century pupil, had filled the pages with marginalia. Schrire didn't know Latin, but he nevertheless grasped the significance of what he was holding. "I wasn't interested in content," he says. "I wanted to understand *how* this student was thinking, not *what* they were thinking about." The annotations were more compelling to him than the text itself.

This discovery set him on a new scholarly quest, which formed the basis for both his MA and doctoral research. Schrire learned Latin and tracked down as many copies of the grammar book as he could: over 200, of which roughly 80 per cent had annotations. He also studied the diaries of schoolchildren and teachers, pedagogical guides and classroom floorplans. Collectively, these texts pointed to a profound shift in culture and cognition, beginning in the 16th century. These social changes were the subject of several of Schrire's published research papers, and will be the focus of his first scholarly monograph.

The cultural shift he writes about coincides roughly with the end of the Middle Ages. Medieval pedagogy, Schrire theorized, was based in rote memorization, with teachers reading aloud to bookless students who were likely standing in rows. But with the advent of the Renaissance, schoolroom practices changed. "Books went from the hands of the teachers to the knees of the students," Schrire explains. Not only did students have their own texts, they also now sat at desks and annotated the pages with conjugation tables, translations and relevant lines from Cicero and Virgil. While medieval students had memorized Latin, their Renaissance successors had internalized both its structure and its literary history. "Renaissance humanism reshaped pedagogy," says Schrire. "Comprehension became the new gold standard of learning."

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In some respects, Schrire's findings were counterintuitive. They implied that, in the medieval era, schoolchildren had somehow kept Latin alive without actually comprehending the language: they often spoke without even knowing what they were saying. His work also broadened our understanding of Renaissance humanism. The shift to a humanistic worldview, Schrire suggested, was as much about cognition as it was about morals.

Schrire's recent findings on numeracy are more contentious, though. They suggest that a cognitive revolution in Europe — the advent and proliferation of modern mathematics — didn't really happen, at least not when we thought it did. As with his previous project, he came across these findings mostly at random. While researching Latin pedagogy, Schrire continued to visit rare books rooms and to arbitrarily call up books. "The thing with PhDs," he says, "is that we always look at the world through a tiny peephole." He wanted to expand this aperture by seeking out texts that had little to do with his immediate interests. Financial documents, he quickly discovered, offer insights into a cohort of people whom historians usually neglect: Europe's ascendent entrepreneurial classes, who were not in the right social class to study Latin in grammar school.

For Reviel Netz, a classics professor at Stanford, it is Schrire's willingness to go beyond his intellectual comfort zone that sets him apart from his peers. "People who study book history usually pick their subject area because they don't like numbers," says Netz. "Ray is travelling into uncharted territory, and he's going to make a wonderful contribution. We need more humanists who aren't scared of mathematics."

Schrire doesn't yet have a thesis for his new project. He wants to visit many more libraries and peruse many more archival books before drawing substantive conclusions. What he does have is a set of tantalizing questions and provisional answers. If, during early modernity, ordinary professionals weren't doing modern math, what were they doing? How did they conceptualize numbers? And how did early capitalism, with its banks, brokerages and trading floors, survive?

One answer to the latter question — which Schrire considers far-fetched but perhaps not so outlandish that it can be ruled out immediately — is that the professional classes were more sophisticated than the historical record would make them out to have been. Perhaps they were doing advanced arithmetic in their heads. A less ennobling but more credible answer is that the early-capitalist period was chaotic, an era in which price signals regularly misfired, suppliers constantly over- or under-produced, lenders sooner or later went bankrupt, and investors and insurers frequently got stiffed.

"Perhaps people simply weren't that rational," says Schrire. "Perhaps they were trying their best and hoping things would work out." Early capitalism, by this account, was a chaotic system sustained by subpar math.

A third answer is that the system functioned passably well because it was somehow better than the sum of its parts. If people were faking knowledge of modern mathematics, perhaps faking was good enough: the mere façade of rationality can be as compelling as the thing itself. This argument at first seems implausible until one applies it to the present day. True, in certain respects, the present really is different from the past. Today, we *actually* practise modern mathematics — primary school students use Hindu-Arabic numerals and high school students write algebraic equations in longhand — although Schrire suspects that these trends took hold later than one might suppose, perhaps in the 19th century. But even if the math of today is more sophisticated than the math of the past, is it really as sophisticated as the commercial and financial systems in which we operate? Or are we fumbling through life, just as our early-modern forebears likely did?

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Over the past decade, for instance, many ordinary people across North America and Europe have made good money in the housing market. But when a person buys a house, do they consider every possible variable to ensure they're getting the right price? Or do they act on instinct and hope for the best? And how many people deeply understand the insurance policies they hold and can say with certainty that they're getting a fair deal? Schrire acknowledges that, like most insurance buyers, he basically does what his advisor tells him to do. "There aren't a lot of rational thoughts going through my mind," he says. "If there were, I would probably cut my insurance in half."

Schrire's work hasn't led him to definitive conclusions, but his initial suppositions are humbling. His research suggests that in early modernity — and maybe in our time, too — rationality is and was a delusion, sustained only by our collective willingness to believe in it.

"Why do people buy and sell?" he asks. "We would like to find a model that accounts for this behaviour. Contrary to conventional wisdom, I'm pretty certain that such a model can't explain capitalism in the past. I'm not even sure it can explain capitalism today." ▲●■

Studying the afterlives of early-modern merchants requires dealing with the material objects they left behind. Bookkeepers used objects like this reckoning token to perform calculations, but these tokens are not well-suited to complex mathematical concepts, and those who used them might not have had a deep understanding of the math they were performing.